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V. Preëminently in English teaching the right of the individual may and should be respected. All teaching in all grades and in all departments is straining towards this goal; but English can take a long step in the right direction at once. It needs not to wait for any experimenter to complete his experiment, or for any committee to publish its report.

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MATHEMATICS IN THE SECONDARY SCHOOLS OF GERMANY, II

In the September number of the *SCHOOL REVIEW* I have sketched the mathematical work of the secondary schools to a point commensurate with the beginning of the American high school course. Following the standard of the Cassel *Realgymnasium* the syllabus is as follows: *

UNTERTERTIA: I. *Geometry*, 2 periods. Theory of the triangle, polygon, and circle. Equality of figures. Constructions. Text-book, *Koppe's Planimetrie*.

II. *Arithmetic*, 2 periods. Reckoning with abstract quantities. Equations of first degree with one unknown quantity. Text-book, *Heis's Sammlung von Aufgaben aus der Allgemeinen Arithmetik*, §1-25; 61-63.

III. *Rechnen*, 1 period. Commercial arithmetic continued.

OBERTERTIA: I. *Geometry*, 3 periods. Reviews. Proportion. Similarity of figures. Relations and contents of rectilinear figures and circles. Pythagorean proposition. Text-book, *Koppe's*.

II. *Arithmetic and Algebra*, 2 periods. Review and extension of work of U III. Theory of proportion, powers, and roots.

* The Weimar course is as follows: *Untertertia*, 5 periods. Commercial arithmetic. Theorems of sum, difference, and product. The circle. Equality of figures. Proportion.—*Obertertia*, 5 periods. Compound division. Factoring. Equations of 1st and 2nd degree with one unknown. Square root. Similarity and measurement of figures. Algebraic geometry.—*Untersecunda*, 5 periods. Plane geometry concluded. Solid geometry. Powers, roots, and logarithms. Equations of 1st degree with more than one unknown quantity and of 2nd degree with one unknown. (Special tasks monthly.)

Equations of first degree with more than one unknown quantity and simple quadratic equations with one unknown. Text-book, *Heis's*.

UNTERSECUNDA: I. *Geometry and Trigonometry*, 3 periods. Elements of trigonometry; computation of triangles. The most important propositions. Solid geometry of the plane and straight line; simple bodies and computation of dimensions, surface, and contents.

II. *Arithmetic and Algebra*, 2 periods. Theory of logarithms with practice in logarithmic reckoning. Quadratic equations. Text-book, *Heis's*.

The geometry of the middle grades presents no striking peculiarities, as I have observed, beyond those of method common to the mathematical instruction in all classes. Euclid seems to find no greater recognition here than in America; the German's explanation is that the great Alexandrian wrote for men, not boys. The four years' course in plane geometry gives ample time not only for thorough grounding in the theory, but also for a variety of practical applications impossible in a shorter course or under a plan which does not provide for simultaneous exercise in arithmetic, algebra, and elementary trigonometry.

The course in arithmetic and algebra is practically a course in *Heis's Collection*. The book seems to be everywhere used, either as text-book or for reference. Since its first appearance in 1837 there have been some ninety editions, in all over 270,000 copies. The influence of this book—the work of a noted mathematician and astronomer—upon the teaching of two generations is clearly demonstrable, and to it may be ascribed many of the peculiarities in German methods. It contains no definitions, no rules and but very few typical solutions. Its strength lies in the systematic gradation of examples and problems, from the rudiments of addition to the involved types of equations of the higher degrees. The world over one will scarcely find four hundred pages of material more suggestive in treatment or richer in detail than this collection for use in the secondary schools of Germany and Austria. There are many other text-books, some of them most excellent, as Bardey's for instance—perhaps better than *Heis's*—but to a striking degree they are all modelled after the master-work. In describing the first 215 pages of the book the work of the

middle grades is described. Its richness and variety is apparent from the presentation in this space of over 2,700 examples, of which some 700 are practical problems. The number actually used by any one class is relatively very small.

The needs of the many who do not continue their schooling beyond *Untersecunda* determine a wider range of study in the middle grades than would doubtless be the case were *Oberprima* the Mecca for all.* Yet at every step the ideal is that theory and practice shall be united; neither factor may be sacrificed to the other. The conceptions underlying the theory of logarithms are not less important than skill in manipulating them; the ability to *think* even a short problem clearly to its conclusion is better than occasional success in finding "the answer" to problems intricate and confusing to the last degree. There is no greater danger in teaching than in overestimating the pupil's strength. Accuracy and facility in calculation and inference depend far more upon right thinking than upon correct writing. The mechanical skill necessary to note properly on paper or blackboard the essential headings of a mathematical demonstration is easily acquired when it is clear what is to be done. It is right thinking, then, that is to be emphasized; correct writing follows as a result. Every task that invites purely mechanical treatment, every problem too difficult for oral analysis, is a step in the wrong direction.

For the upper classes of the Cassel *Realgymnasium* the following courses are prescribed: †

OBERSECUNDA: I. *Geometry and Trigonometry*, 3 periods. Plane trigonometry and plane geometry reviewed and concluded. Solid geometry. Practical applications. Text-books, *Koppe's*; *Gauss' Logarithmic Tables*.

* Of 20,038 Prussian secondary pupils that left school in 1889-90, 4,105 completed the course, 8,051 went out at the end of U II, and 7,882 from the lower classes.

† Weimar course: *Obersecunda*, 5 periods. Solid geometry. Equations of 1 and 2 degree with more unknowns. Determinants. Arithmetical and geometrical series. Compound interest and annuities. (Special tasks monthly)—*Prima*, 5 periods. Permutations and combinations. Chance. Binomial theorem. Series—exponential, logarithmic, sine and cosine. Analytic geometry of the plane. (Special tasks monthly.) Perspective drawing and shading. Map projection.

II. *Arithmetic and Algebra*, 2 periods. Arithmetical and geometrical series. Compound interest and annuities. Quadratic equations with more unknowns. Permutations and combinations, binomial theorem applied to positive whole exponents. Text-book, *Heis's*.

UNTERPRIMA: I. *Geometry and Trigonometry*, 3 periods. Solid Geometry continued. Theory of plane and spherical angles. Spherical trigonometry and its application to mathematical geography. Conic sections. Text-book, *Koppe's*.

II. *Arithmetic and Algebra*, 2 periods. Continued fractions and applications. Arithmetical series of second order. Cubic equations. Problems of *maxima* and *minima*. Reviews. Text-book, *Heis's*.

OBEPPRIMA: I. *Geometry*, 3 periods. Solid geometry reviewed and concluded. Analytic geometry. Problems in mathematical geography. Geometrical drawing.

II. *Arithmetic and Algebra*, 2 periods. Functions and applications to higher equations, especially those of third degree. Exponential, logarithmic and sine and cosine series. Practical applications. Text-books by *Koppe*, *Heis*, and *Gauss*.

It is not my purpose to dwell on details which may be found in practice in any good American school. For this reason the mere statement of what is done in the upper grades is sufficient. It is of more interest to know in how far these courses are actually followed and with what success. And here's the rub. No one school, no single type of schools, can adequately represent the work of all, and yet a norm of some kind is necessary. The *Oberrealschulen* are too few in number to set a standard—and besides their material is not the best; the ambitious parent is prejudiced in favor of the *Gymnasium*, the door of all that is desirable in civil and professional life, but if the boy cannot get on there the *Realgymnasium* and *Realschule* are next on the list—and on the other hand many classical schools make a farce of mathematics, the better the classics the worse for mathematics. The gymnasial course is concluded with quadratics and binominal theorem, the elements of solid geometry and plane trigonometry. But other things equal, the demands on the gymnasial master are as great as in the *Real*-schools owing to the restricted number of recitations—four a week, save in *Tertia*, where only three are given. To overcome this difficulty some teachers prefer to divide the time of *each* recitation equally between algebra and geometry. The objections are at once apparent. But ham-

pered as the gymnasial teachers are in many ways, I have been surprised to see how successfully the best of them master their environment. I am convinced, however, that the conditions which determine the highest results in mathematics are to be found in the *Realgymnasium* of non-Prussian states.

Pupils are promoted from class to class on the advice of the teacher. He has for his guidance the marks of the regular recitations, the pupils' note-books, and the monthly tasks done at home. Written examinations are held in some schools about once a month, the regular recitation period being used for the purpose, but so far as I have learned the practice is not general. The final examination of the course is both written and oral,* the latter being held by the master in the presence of a committee of his colleagues and the Inspector, *Oberschulrat*, of the province. The papers from the written test also after being read and marked ("very good, good, satisfactory, or unsatisfactory") by the master are forwarded to the inspector. In both written and oral examination mathematics is considered a main subject. In *Real* schools the test is undoubtedly efficacious and the results accepted as they stand, but everywhere gymnasial teachers are

* I give herewith the final written tests for the *Realgymnasien* of Cassel and Weimar at Easter, 1894 :

CASSEL: *Time, five hours*—(1) A sphere with radius $r=15$ is cut by a plane a into two parts so that the entire surface of the one holds the relation to the other of $m:n=3:2$. What is the height of the smallest part? (2) Two stars appear to be at a distance of $17^{\circ} 15'$ from each other. The declination of the one is $21^{\circ} 9'$, of the other $35^{\circ} 8'$; what is the difference in right ascension between the two? (3) A parabola and a straight line have the equations, $y^2=4x$ and $y=x-3$. How large is the segment of the parabola which is intersected by the straight line? (4) $x=3+\sqrt{\frac{10}{x}}+2$.

WEIMAR: *Time, six hours*.—(1) Find a circle touching two given circles, one of them in a given point. (2) Given the longest side of a triangle equal to 25 cm, and the ratio $2:3:5$ of the radii of the three circles touching externally, find the other two sides, the angles and the radius of the inscribed circle. (3) A cone made of pine wood (sp. gravity $\frac{1}{2}$), whose radius and altitude are the same, has had $\frac{1}{8}$ of its volume cut off at the top. How far does it, with its base turned upward, go down in water? (4) Two ellipses, whose minor axes are as $1:\sqrt{2}$, have each the end of its major axis coinciding with the centre of the other. Find the coördinates of their points of intersection, and the condition of their being of the same length.

outspoken against the slight put upon their efforts under cover of these examinations. I am told that it really makes little difference whether or no a boy is proficient in mathematics if he knows his classics. The absence of an exact marking system forces the teacher of mathematics to join issues of judgment with his philosophical colleagues; in questions of pupils' maturity his opinion has small weight. The average boy is not slow to take advantage of all that comes his way. Indirectly, therefore, the best interests of a department are jeopardized; weakness is introduced where there should be strength.

Not only have examinations no terrors, but it is not generally recognized that a knowledge of mathematics increases one's chances of success in professional life. These factors all tend to make a complicated problem the more involved. And not all masters are teachers. The results in such cases are deplorable in any event, but especially so under German methods. When all depends upon the teacher and he fails, what is to become of the pupil! He has no incentives ideal or material to spur him on, and if he be a genius the very text-books discourage independence. The pupil must follow; he can neither lead nor go alone.

The German ideal is very high, though possibly none too high. But for ideal methods you must have ideal teachers, and no one will say that Germany has too many of such. Indeed, I am told that it is more difficult to find *good* teachers of mathematics than of any other subject. One cause is not far to seek. The German universities are founded on the classics. For three centuries, and more, a knowledge of Greek and Latin has been the *conditio sine qua non* of admission to university privileges. And to-day only graduates of the *Gymnasien* have full rights. Popular demands of the last few years have secured some concessions for the *Real* schools, but in each step the intellectual monopolists have fancied an encroachment on the dignity of learning. The medical fraternity, as individuals, would be glad of more and better training in science and the modern languages, but the pride of the profession demands Greek and Latin instead. What wonder then that questions of caste have arisen from the partizan strife over the rights of the secondary schools! It even enters the university and casts a shadow upon those courses fit only for *Real* school graduates. And what are these courses? *Mathematics* and *Nat-*

*ural Sciences!** The moral is plain. The teacher of mathematics has not the social standing of the philologist. Caste enters into the school life and the students see it. Nothing but the force of personality can break these bonds—a personality stronger than most men have. Or stated in another way, there is nothing inherent in the office of the mathematician to gain for him a social rank equal to that indissolubly connected with the humanistic studies. I speak especially of the mathematical post in the *Gymnasium*; in the *Real* school there may not be the same internal gradations but the school itself stands on a lower level. The outlook discourages the best candidates and as a result the mantle too often falls on men who have not the keen, clear-cut intellects so essential to success under the German system.

Despite these defects in particulars, there are excellencies in the German plan which no thoughtful teacher, no one charged with school management, can afford to disregard. The teacher is always a *trained specialist*. No man finds a post in a German school nowadays who is not master of what he purposes to teach; he has had, moreover, two years of professional training in theory and practice. And on the other side there are (1) the unity of the course, which is rendered still more effective by the interlacing and blending of the subordinate lines, (2) the emphasis put upon mental operation, and (3) the supreme end towards which all aims—*logical thinking*—these at least can be unreservedly commended. As for the German methods, the final question is, Does the pupil become an *independent* thinker? Granting good teachers my answers are, *No*—so far as the poorest are concerned; *Very doubtful*—for the average; but emphatically *Yes*—for the best in the class. To the earnest student the very consciousness of increasing strength and clearness of vision is of itself an incentive to mastery. He feels that he is getting something better than skill in the manipulation of symbols. The external operation is for him merely the record of his own thought, convenient and useful for the time being, but relatively unessential. "The life is more than meat, and the body is more than raiment."

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* Realgymnasiasts may also study the modern languages.